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Effect of nuclear deformation in proton radioactivity

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Abstract - The separation energies are calculated for the low and medium mass drip line nuclei in the periodic table. The mass excess values for the nuclei are taken from Wang, Audi-Wapstra mass table (Wang et al., 2012). The possible proton emitters are identified using this separation energy calculation as $S_p < 0$. The shape of the single proton emitters are calculated using deformation parameters applying tuned Cranked Nilsson model with Strutinsky shell correction approach in the chosen drip line nuclei. Odd Z are found to be good proton emitters and much deformed than their even Z neighbors. This is due to the reason that pairing correlations are strongly reduced in odd Z nuclei and as a result the nucleus is driven towards larger deformation. For nuclei, far from stability, have significant shape deformation in their ground state. This is reflected in the calculations of quadrupole deformation parameter. Nuclear structural changes are analyzed and decay probabilities are calculated. Obtaining more precise data on deformed and nearly spherical proton emitters will help one to test the models of proton radioactivity decay rates.

Index Terms - Cranked Nilsson Strutinsky shell correction method, Potential Energy Surface, Proton radioactivity, Quadrupole deformation, Shanmugam Kamalaharan model.

1 INTRODUCTION

Study of proton and neutron drip line nuclei are of utmost importance in nuclear structure physics. Proton emission from spherical and deformed parent nuclei supplies information about these unstable nuclides. This observation is mandatory to maintain spectroscopic data about these nuclides at the limits of stability. At the same time, this study also kindles strong test to the theoretical models on proton radioactivity. Because of the Coulombic repulsive force, proton drip line nuclei are studied vastly up to Z=91 when compared to neutron drip line. Proton emission is studied theoretically using quantum mechanical tunneling assuming a spherical shape for the parent nuclei. However the experimentally observed proton emitters in the light and medium mass region are found to be deformed. It is very important to study and analyze this proton emission from deformed parent nuclei. A lot of interest is shown by different experimentalist and theoreticians in studying the neutron deficient nuclei in the medium mass region [1,2]. Experiments done at Daresbury and Munich Labs [3,4] on Iodine and Cesium nuclei supported that the parent nuclei are not spherical. Further experiments performed by different groups gave a strong confirmation for the existence of deformation in the light and medium mass nuclei [5,6,7].

In the present paper, we theoretically investigated the deformation of proton rich nuclei using Shanmugam Kamalaharan model (SK model) [8,9] and cranked Nilsson model [10,11]. Proton emitting nuclides with their $53<Z<61$ are deformed in their ground state and their study gives a confirmation for theoretical explanation regarding proton radioactivity. Calculations are performed in this region for Cs, La, Pr, Pm isotopes, in which appreciable deformations are observed and are compared with reference values.

2 THEORETICAL FRAMEWORK

The present investigation has been focused on the new radioactivity related with microscope structure of three dimensional rotations in the cranking approximation. The model used contains three main building blocks that govern the ground state structure of chosen nuclides i.e., proton separation, deformation and shape of the nuclides. Here we have chosen the SK model which was formulated by Shanmugam and Kamalaharan for exotic radioactivity [8]. This is extended to proton radioactivity and as an advanced step, the deformation of the parent nuclei is incorporated in calculating the life-times of the proton emission. Here the height of the barrier is determined with accuracy by taking explicit values for zero point vibration energy. The value of the potential barrier at the top is made to coincide with the experimental value by considering finite range Yukawa plus exponential potential. Q-values are used to include the shell effects at the ground state [12,13]. The shape of the barrier in the overlapping region which connects the ground state and the contact point is approximated by a third-order polynomial suggested by Moller and Nix [14]. The post scission region is considered to have Yukawa plus exponential potential to incorporate the redistribution of charges among the fragments in the region between the sharp contact and the point up to which the finite range effects persist.

The separation energy of the last proton is the energy required to separate one proton from a nucleus. The separation energies are calculated for the low and medium mass nuclei in the periodic table. The mass excess values for the nuclei are taken from Wang, Audi-Wapstra mass table [15]. The possible proton emitters are identified using this separation energy calculation as $S_p < 0$. These nuclei are in a ready state to give out a proton enabling them to reach a stable configuration. Half-lives of the identified proton emitters are calculated using SK model. The proton emitters are considered to have a deformed shape and the deformation parameters are calculated using cranked Nilsson Strutinsky shell correction method [16]. Potential Energy surface plotting will help in determining the shape changes shown by the proton emitters. This will enable us to check the theoretical models used in our calculations.

2.1 SK Model on Proton Radioactivity

Half Life is unique for an isotope and is considered to be the time taken by the parent nucleus to be transformed to its daughters. So $t = t_{1/2}$ at N = N_0 / 2. Therefore,

$$t_{1/2} = \frac{0.693}{\lambda} \tag{2.1}$$

Half-life can take any value between few microseconds to several billion years. An unstable nucleus leads to more than one decay. The half lives of proton radioactivity are studied using SK model. For the post scission region, one can use a finite range Yukawa plus Exponential potential along with the Coulomb potential and a third order polynomial can be used for the overlapping region. The original shape of the barrier is not known and it is wise to take a parabolic nature near the top which should have a local minimum corresponding to ground state configuration. A cubic potential is the appropriate shape is the simplest form which meet these physical requirements. The centrifugal barrier can be neglected in cluster radioactivity and contributes appreciably in the case of alpha decay. For proton emission the contribution of centrifugal effect becomes very much significant. The effect of centrifugal barrier is incorporated in the post scission region for proton radioactivity.

2.2 Inclusion of Deformation Effect

The parent nucleus is considered to be split into two uneven daughters. Also the parent is considered to be deformed and it is incorporated in the calculation spheroidal deformation β_2 and Nilsson's hexadecapole deformation β_4. Now the heights of the uneven daughters that is one heavy and one light portion are taken to be respectively,

$h_1+h_2 = 2a_0$

where a_0 is the semi major axis of a prolate spheroidal parent nucleus with its major axis parallel to the fission direction,

$$a = R_0\left[1+\left(\frac{5}{4\pi}\right)^{1/2}\beta_2+\left(\frac{9}{4\pi}\right)^{1/2}\beta_4\right] \tag{2.2}$$

$$\frac{3h_1}{a_0}+\left(1-\frac{h_1}{a_0}\right)^3-1=\frac{4A_1}{A} \tag{2.3}$$

where R_0 is the sharp surface radius of equivalent spherical nucleus. Solving the above equations one can get the values of h_1 and h_2. The distance r_i between the centers of mass of two daughter fragments in the deformed parent nucleus is now given as

$$r_i=\frac{3}{4}\left[\frac{h_1^2}{a_0+h_1}+\frac{h_2^2}{a_0+h_2}\right] \tag{2.4}$$

The deformation parameters are taken from Moller et al., (2001).

2.3 Inertial Mass Coefficient

The nuclear inertial mass coefficient $B_r(r)$ is considered to depend on deformation and the expression for the inertia in the model of Moller [14] is

$$B_r(r)-\mu = f(r,r_t)k\left(B_r^{irr}-\mu\right) \tag{2.5}$$

where

K = 16, $r_t = a_2 + R_1$ and

$$f\left((r,r_t\right)=\begin{cases}\left|\dfrac{r_t-t}{r_t-r_i}\right|^4 r\le r_t \\ 0 \quad\quad r>r_t\end{cases}$$

Here, B_r^{irr} is the inertia whose numerical results are approximated by Moller and Nix as

$$B_r^{irr}-\mu=\frac{17}{15}\mu \quad \exp\left[-\frac{128}{51}\left(r-\frac{3}{4}\right)\right] \tag{2.6}$$

In the case of asymmetric fission of a deformed parent nucleus, one can take

$$B_r^{irr}-\mu=\frac{17}{15}\mu \quad \exp\left[-\frac{128}{51}\left(\frac{r-r_t}{a_0}\right)\right] \tag{2.7}$$

where a_0 is the semi major axis of the prolate spheroidal parent nucleus.

2.4 Lifetime Calculation

Expressing the energies in MeV, lengths in fm, and time in seconds, for calculating the lifetime, the formula $T = \frac{1.433 \times 10 - 21}{E_v}\left(1 + \exp K\right)$ is considered.

The action integral K including left and right integral is given to be

$$K = \frac{2}{\hbar}\left[\int_{r_a}^{r_1}\left[2B_r(r)V(r)\right]^{1/2} dr + \int_{r_t}^{r_b}\left[2B_r(r)V(r)\right]^{1/2} dr\right] \tag{2.8}$$

The decay width is calculated using the relation

$$\Gamma = \hbar/\tau \tag{2.9}$$

where τ is the mean life time of the nucleus.

Newton-Raphson method is employed to find the limits of integration r_a and r_b which are the two appropriate limits of the integral.

$$r_b = \frac{\mu Z_1 Z_2 e^2 + \sqrt{\left(\mu Z_1 Z_2 e^2\right)^2 + 2\mu Q l(l+1)\hbar^2}}{2\mu Q} \tag{2.10}$$

where μ is the reduced mass given by $\mu = m\frac{A_1 A_2}{A_1 + A_2}$

Here m is the nucleon mass. A_1 is the mass of the daughter and A_2 is the mass of the emitted nucleus. After incorporating the centrifugal barrier contribution, the interaction potential is given as

$$V(r) = \frac{Z_1 Z_2 e^2}{r} + \frac{\ell(\ell+1)}{2\mu r^2} + V_n(r) \tag{2.11}$$

$$V(r) = \frac{Z_1 Z_2 e^2}{r} + \frac{l(l+1)\hbar^2}{2B_r(r)r^2} + V_n(r) \tag{2.12}$$

where the first, second and third terms on the right hand side are the Coulomb, centrifugal and finite range potentials respectively.

The proton ejected from the parent nucleus is considered to be subjected to finite range effects. So finite range effects are included in the post-scission potential in our calculation. This is not found in other models used to explain proton radioactivity. The nuclear inertia $B_{r(r)}$ represents the motion along the fission direction. The post scission barrier potential is modified with finite range effect, deformation parameter of the parent and the daughter nucleus. The barrier potential is taken to be Yukawa plus exponential potential in the outer region, and a narrow cubic potential form as given by Nix in the inner region.

3. TRIAXIALLY DEFORMED CRANKED NILSSON STRUTINSKY SHELL CORRECTION METHOD

To find the deformation in the ground state as well as the excited state of the nucleus can be studied using deformed shell model, otherwise called the cranked Nilsson model. It also gives the theoretical framework for obtaining potential energy surfaces of the considered nuclei as a function of deformation β and non-axiality γ parameters at different spins by the Strutinsky shell correction method.

In the Nilsson model, the potential in Hamiltonian comprises the anisotropic harmonic oscillator potential plus the spin-orbit and centrifugal potentials. For a zero spin nuclei shell energy calculations takes a single particle field

$$H_0 = \Sigma\, h_0 \tag{3.1}$$

here h_0 is the triaxial Nilsson Hamiltonian given by

$$h_0 = \frac{p^2}{2m} + \frac{1}{2} m \sum_{i=1}^{3} \omega_i^2 x_i^2 + C\, l.s + D\left(l^2 - 2\langle l^2 \rangle\right). \tag{3.2}$$

By Hill- wheeler parameterization the three oscillator frequencies ω_i are given as

$E_k = \hbar\omega_k = \hbar\omega_{GDR} \exp [-\sqrt{5/4\pi} \beta \cos (\gamma - 2/3 T_i K)]$ with the constraint of constant volume for equipotentials

$$\omega_x \omega_y \omega_z = \mathring{\omega}_0^3 = constant \quad (3.3)$$

The value for $\hbar\omega_0$ is taken as

$$\hbar\omega_0 = \frac{45.3\,MeV}{(A^{1/3} + 0.77)} \quad . \quad (3.4)$$

Upto N=11 shells, the Hamiltonian is diagonalized in cylindrical representation using the matrix elements.

In a non zero spin (rotating) nucleus ($I \neq 0$) without internal excitation, the nucleons are considered to move in cranked Nilsson potential with the deformation defined by β and γ. The cranking is assumed along the Z-axis and the cranking frequency is ω. So the Hamiltonian for such a case is given by

$$H_\omega = H_0 - \omega J_z = \sum h_\omega \quad (3.5)$$

where $$h_\omega = h_0 - \omega j_z . \quad (3.6)$$

Diagonalization of

$$\hbar_\omega \phi_i^\omega = e_i^\omega \phi_i^\omega \quad (3.7)$$

gives the single particle energy e_i^ω and wave function ϕ_i^ω.

The evaluation of total energy for large deformations is done through the summation of single particle energies and the discrepancies faced there is removed in the Strutinsky shell correction method by suitably tuning [11] the angular velocities to yield fixed spins. For unsmoothened single particle level distribution the spin I is given as

$$\mathrm{I} = \int_{-\infty}^{\lambda} g_2 de^{\omega} = \sum_i \langle m_i \rangle \tag{3.14}$$

and
$$\mathrm{E}_{sp} = \int_{-\infty}^{\lambda} g_1 e^{\omega} de^{\omega} + \hbar\omega \mathrm{I} = \sum_i e_i^{\omega} + \hbar\omega\mathrm{I}\,. \tag{3.15}$$

In the tuning method the total spin is adapted and is calculated as

$$\mathrm{I} = \tilde{\mathrm{I}}_z = \sum_{\nu=1}^{\mathrm{N}} \left\langle \tilde{J}_z \right\rangle_{\nu}^{\omega} + \sum_{\pi=1}^{\mathrm{Z}} \left\langle \tilde{J}_z \right\rangle_{\pi}^{\omega} \quad . \tag{3.19}$$

For a chosen integer or half integer spins the above relation permits to select numerically the ω values. The calculations are repeated accordingly as the frequency values $\omega(I)$ change from one deformation point to another.

The Strutinsky method [16] of shell correction has been successfully used in calculations of total energy as a sum of smooth liquid drop energy and oscillating shell correction energy. The total energy is given by

$$E_T = E_{RLDM} + \left(E_{sp} - \tilde{E}_{sp}\right) \tag{3.20}$$

where
$$E_{RLDM} = E_{LDM} - \frac{1}{2} I_{rig}\omega^2 + \hbar\omega\tilde{I} \quad . \tag{3.21}$$

The liquid drop energy E_{LDM} is given by the sum of Coulomb and surface energies as

$$E_{LDM}(\beta,\gamma) = \left[2\chi\left(B_c - 1\right)a_s + \left(B_s - 1\right)\right] \tag{3.22}$$

where B_c and B_s are the relative Coulomb and surface energies of the nucleus. The values used for the parameters a_s and χ are a_s = 19.7 MeV and fissility parameter $\chi = (Z^2/A)/45$ where Z and A are the charge and mass numbers of the nucleus.

The rigid body moment of inertia I_{rig} is defined by β and γ including the surface diffuseness correction and $\tilde{I}$ is the Strutinsky smoothened spin [16]. For an ellipsoidal shape described by the deformation parameter β and shape parameter γ, the semi axes R_x, R_y, R_z are given by,

$$R_x = R_0 \exp\left[\sqrt{\frac{5}{4\pi}}\beta\cos\left(\gamma - \frac{2\pi}{3}\right)\right]$$

$$R_y = R_0 \exp\left[\sqrt{\frac{5}{4\pi}}\beta\cos\left(\gamma - \frac{4\pi}{3}\right)\right]$$

and $$R_z = R_0 \exp\left[\sqrt{\frac{5}{4\pi}}\,\beta\cos\gamma\right] .$$

By volume conservation we have

$$R_x R_y R_z = R_0^{0\,3} \tag{3.23}$$

where R_0^0 is the radius of the spherical nucleus. Here,

$$\mathrm{R}_0^0 = \mathrm{r}_0 \mathrm{A}^{1/3} \quad (\mathrm{r}_0 = 1.16\,\mathrm{fm}) . \tag{3.24}$$

The moment of inertia about the Z axis is given by

$$\frac{I_{rig}(\beta,\gamma) + 2Mb^2}{\hbar^2} = \frac{1}{5}\frac{AM\left(R_x^2 + R_y^2\right)}{\hbar} + \frac{2Mb^2}{\hbar^2} \tag{3.25}$$

where $2Mb^2$ is the diffuseness correction to the moment of inertia and the diffuseness parameter b = 0.90 fm [17]. The zero temperature potential energy surfaces for fp shell region isotopes have been obtained by the tuned Strutinsky procedure [18]. In these calculations *ground state spin* is considered, also without temperature (0.0 MeV), γ from -180° to -120° in steps of 10°, and β from 0.0 to 0.8 in steps of 0.1. The Hill Wheeler expressions for the frequencies have been used in the cranked Nilsson model.

4 RESULTS AND DISCUSSIONS

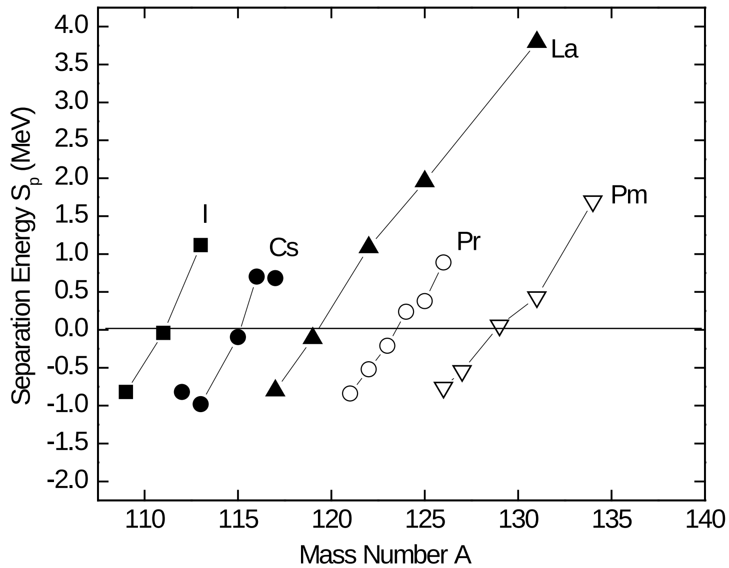

Fig. 1. One Proton Separation Energy vs. Mass Number of the parent nucleus

In this work, separation energies are calculated for different proton numbers Z for fixed values of N = 3, 6, 8, 10, 11, 12, 14, 16, 18, 20, 22 … and different neutron numbers for various fixed Z values. The mass value for the parent and the daughter nuclei are taken from the Wang, Audi-Wapstra mass table [15]. The separation energy usually gives information about stability and magicity in the light nuclei. Also the probable one proton emitters are identified from their separation energy values since the one proton emitters should have negative S_{1p} values.

Fig. 1 shows a plot of the proton separation energies for the known ground-state proton emitters of the odd-Z nuclides in the region 53< Z< 61. Table 1 gives the list of probable one proton emitters with their separation energy values.

TABLE 1

ONE PROTON SEPARATION ENERGY VALUES FOR THE ONE PROTON EMITTERS

Nucleus	**Separation energy (MeV)**
I^{109}	-0.821
I^{111}	-0.041
Cs^{112}	-0.821
Cs^{113}	-0.981
Cs^{115}	-0.097
La^{117}	-0.801
La^{119}	-0.111
Pr^{121}	-0.841
Pr^{122}	-0.521
Pr^{123}	-0.211
Pm^{126}	-0.761
Pm^{127}	-0.541

For heavy nuclei separation energy is almost nearing 1 MeV. In such heavier nuclei, more than one proton emitters are found to exist for the same element.

To study ground state proton emitters, one must look at nuclei with half lives in the range of 1 μs-1 s, to which current experimental capabilities are designed to handle. This has encouraged people in looking at heavier elements (Z > 50) [19, 20]. The identified one proton emitters in the chosen region are analysed for their decay with their calculated half-lives, lifetime and decay width. This is performed using SK model designed for proton radioactivity.

TABLE 2

ONE PROTON EMITTERS WITH THEIR HALF-LIVES

Nucleus	**$T_{1/2}$ (s)**	**Lifetime (s)**	**Decay width (eV)**
I^{109}	1.26×10^{-5}	1.818×10^{-5}	3.605×10^{-11}
I^{111}	1.25×10^{81}	1.81×10^{81}	3.63×10^{-97}
Cs^{112}	5.188×10^{-5}	7.48×10^{-5}	8.756×10^{-12}
Cs^{113}	5.22×10^{-7}	7.536×10^{-7}	8.73×10^{-10}
Cs^{115}	1.65×10^{45}	2.39×10^{45}	2.76×10^{-61}
La^{117}	3.155×10^{-3}	4.55×10^{-3}	1.437×10^{-13}
La^{119}	5.610×10^{42}	8.09×10^{42}	8.133×10^{-59}
Pr^{121}	1.25×10^{-3}	1.816×10^{-3}	3.606×10^{-13}
Pr^{122}	3.4119×10^{4}	4.922×10^{4}	1.33×10^{-20}
Pr^{123}	2.831×10^{24}	4.08×10^{24}	1.612×10^{-40}
Pm^{126}	0.213	0.308	2.136×10^{-16}
Pm^{127}	7.311×10^{4}	1.05×10^{5}	6.26×10^{-21}

From Table 2 it is noticed here that nuclides like I^{109} $Cs^{112,113}$ La^{117} Pr^{121} Pm^{126} are found to be unstable with their short half-lives. Also their separation energy of the last proton is very less, they are said to undergo decay through one proton emission. Fission becomes more active and alpha decay half-lives become too short that proton emission is favored in this region. If the decay width is larger, the decay from that state has more probability to take place and so resulting in shorter half-lives. Broader the energy states, lesser the value of half-life for the proton emitters. So this decay width calculation enables one to identify the efficient proton emitters [19, 20]. The observation of these and other proton emitters expected to be found in the $53 < Z < 61$ region should greatly increase the understanding of the role of deformation in the proton decay process.

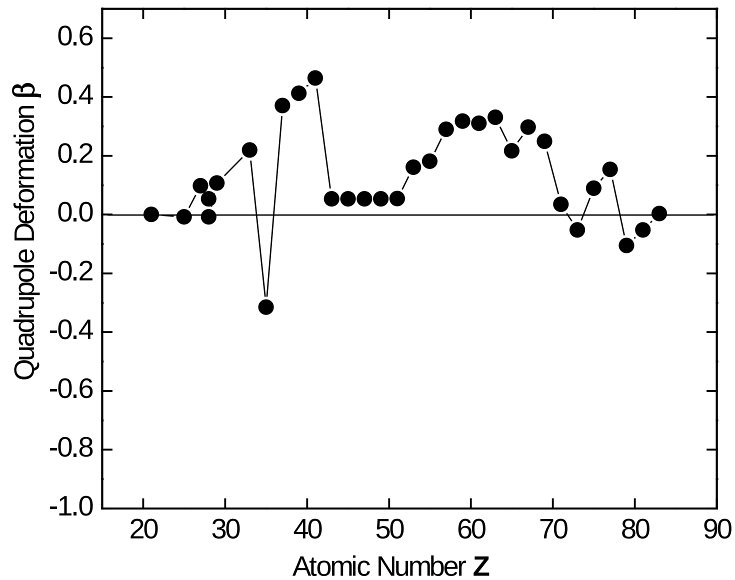

Fig. 2. Quadrupole Deformation as a function of Atomic Number of the proton emitters

Fig.2. depicts the quadrupole deformations of odd Z nuclei in the region 53 < Z < 80. This entire chain of proton emitters is observed to have both prolate and oblate shape. This will help us in knowing the structural information and hence their decay channels. The array of theoretical tools available for deformed emitters is not well developed [21]. The existing ones fall into three general categories. The first family of calculations is based on the reaction theoretical framework of unified fission model. The second uses the theory of Gamow (resonance) states. Finally an approach based on the time dependent Schrodinger equation has been introduced by Talou [22]. The calculated ground state quadrupole deformation of the nuclei in 53 < Z < 61 is shown in Table 3. Odd Z are found to be good proton emitters and much deformed than their even Z neighbours. This is due to the reason that pairing correlations are strongly reduced in odd Z nuclei and as a result the nucleus is driven towards larger deformation. Much stronger pairing in even Z

nuclei results in almost spherical shapes. The relatively high potential barrier enables the observation of ground state proton radioactivity less possible in the region $Z \leq 50$.

Any deformation from the spherical shape is because of the competition between the Coulomb and surface energies [23]. Coulomb force repels the protons to stay far and favors deformation. Since the Coulomb force is not so strong to increase the nuclear volume and create more surfaces among the protons. This increases the surface energy by increasing the surface area. The surface tension force balances the Coulomb force to keep the surface area a minimum. The surface energy tries to preserve the spherical shape. This surface energy domination is found in light nuclei making them spherical. As Z increases, the Coulomb repulsion increases favoring deformation [24]. As Z increases, the number of valence nucleons outside the closed shell increases. This decreases the vibration energy as they become less compact. But the interaction between the valence nucleons and closed core brings about slight deformation [25]. For nuclei which are not far from stability line, this is the case.

Our deformation values are compared with the values given by Moller et al [26] and it is found to be in agreement with those values. The deformation is found to take a non zero value in the chosen region. This shows that calculating the half-lives with spherical shape will not yield considerable value that will agree with experimental values. The calculated half-lives affect the angular momentum of the Nilsson orbitals and so this deformation calculation will help to carry out the structural studies of various nuclei known for proton emission. This deformation related proton separation phenomenon which had earlier been noted in the calculations of heavy nuclei, is now seen to be a more common feature of nuclides near the proton drip line and this work will allow for more reliable predictions of the properties of exotic nuclei.

The scope for discovery of new proton emitters is now limited. Competition from fission becomes overwhelming as one tries to produce drip line nuclei with Z>83 with fusion evaporation reactions because they create the compound nuclei with high angular momentum. At lower masses, the Q-window for observable proton decay is very small and it becomes increasingly unlikely with decreasing Z that a particular element will have a proton-unstable isotope with a half-life in the range accessible with current separator techniques ($1\mu s \leq t_{1/2} \leq 1s$). However there are still gaps to be filled.

These deformation studies on proton emitters offer insight into proton binding and the evolution of shell structure at the edge of stability line. A chain of proton emitting nuclei from Z=53 to Z=61 is established. The data have revealed a systematic trend in proton emission spectroscopic factors that are described using SK model and Cranked Nilsson Strutinsky shell correction method. The variations in the calculated values of β when compared with Moller [26] calculations are found to be negligibly small.

TABLE 3

THE CALCULATED QUADRUPOLE DEFORMATION VALUES ON COMPARISON WITH THE MOLLER VALUES

Nucleus	**Quadrupole deformation β_2**	**Octupole deformation β_6**	**Quadrupole deformation β_2 (Moller et al., 2001)**	**Octupole deformation β_6 (Moller et al., 2001)**
I^{109}	0.152	0.06	0.16	0.060
I^{111}	0.203	0.05	0.161	0.051
Cs^{112}	0.222	0.06	0.208	0.067
Cs^{113}	0.183	0.05	0.207	0.052

Cs^{115}	0.252	0.07	0.252	0.075
La^{117}	0.251	0.06	0.290	0.100
La^{119}	0.304	0.04	0.299	0.076
Pr^{121}	0.302	0.050	0.318	0.075
Pr^{122}	0.303	0.046	0.319	0.066
Pr^{123}	0.303	0.045	0.319	0.049
Pm^{126}	0.252	0.01	0.310	0.020
Pm^{127}	0.321	0.012	0.311	0.011

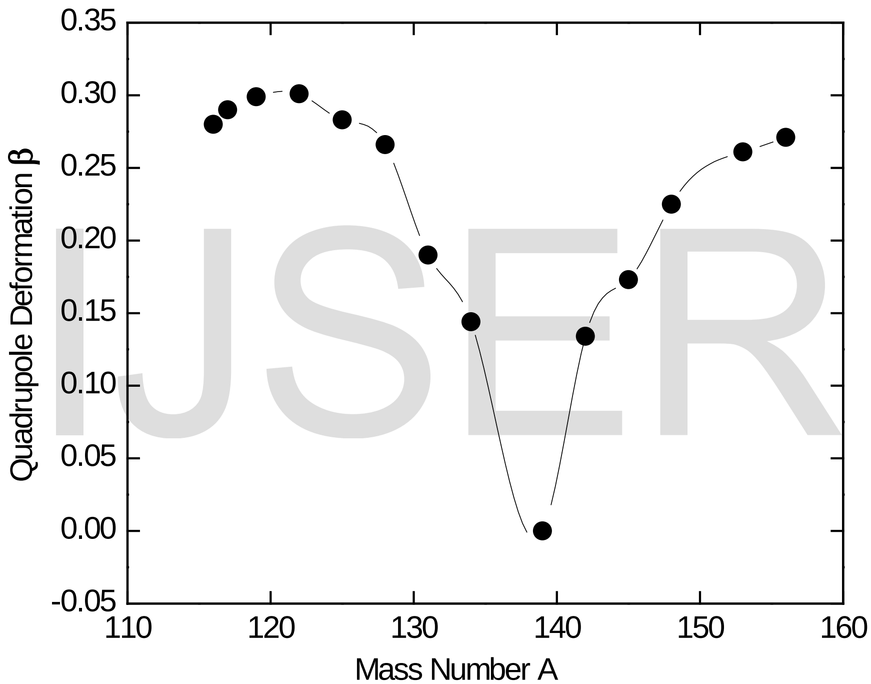

Fig. 3. Deformation Parameter as a function of Mass number for La isotopes

Fig. 3 shows the deformation values plotted for La isotopes. Quadrupole deformation values show a significant variation among the isotopes. Lower mass isotopes are found to be highly deformed and at A= 139, the corresponding isotope is found to take a spherical shape. Further the shape is found to be deformed in the higher mass region. The known proton emitters of La isotopes $La^{117,119}$ have significant deformation with $\boldsymbol{\beta_2}$ = 0.252 and 0.304 respectively.

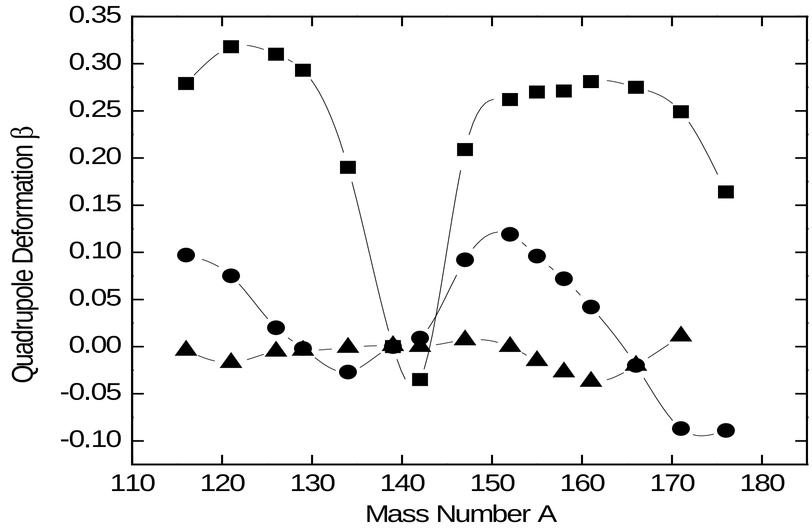

Fig. 4. Deformation Parameter as a function of Mass Number of Pr isotopes

Fig. 4 shows the variation of deformation parameters for Pr isotopes. The minimum is observed at A=141 and further a dip expected in the higher mass region. Still the know proton emitters of Pr isotopes $Pr^{121,122,123}$ have significant deformation with $\boldsymbol{\beta_2}$=0.302, 0.303, 0.303 respectively. The hexadecapole and octapole deformation values are indicated by solid triangles and circles respectively. They are found to vary slowly with increase in the mass of the isotopes. These values are not found to show any significant effect on the shape of the isotopes.

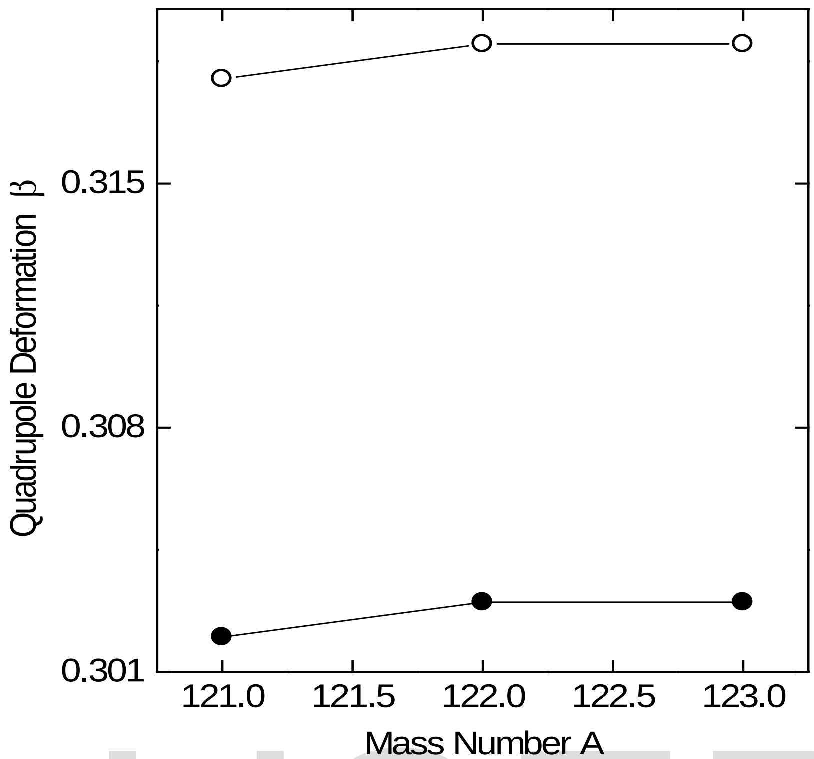

Fig. 5. Quadrupole Deformation values calculated (dark circle) for proton emitters of Pr isotopes in comparison with Moller et al values (open circle)

Fig. 5 shows the quadrupole deformation values as a function of mass number for Pr isotopes. Our calculated values are represented by dark circles and are compared with Moller et al values shown by open circles. Significant shape change is not observed in the Pr isotopes and our calculation does not yield a closed coincidence deformation value as given by Moller. This may be due to the potential shape taken for our calculation. Fig 6 shows the variation of deformation parameters for Pm isotopes. The dip is found at A=140 and later at A=181. In the medium mass region, appreciable deformation is observed. The hexadecapole and octapole deformation values are indicated by solid triangles and open circles respectively The proton emitters $Pm^{126,127}$ show a higher deformation value of 0.252 and 0.321 respectively.

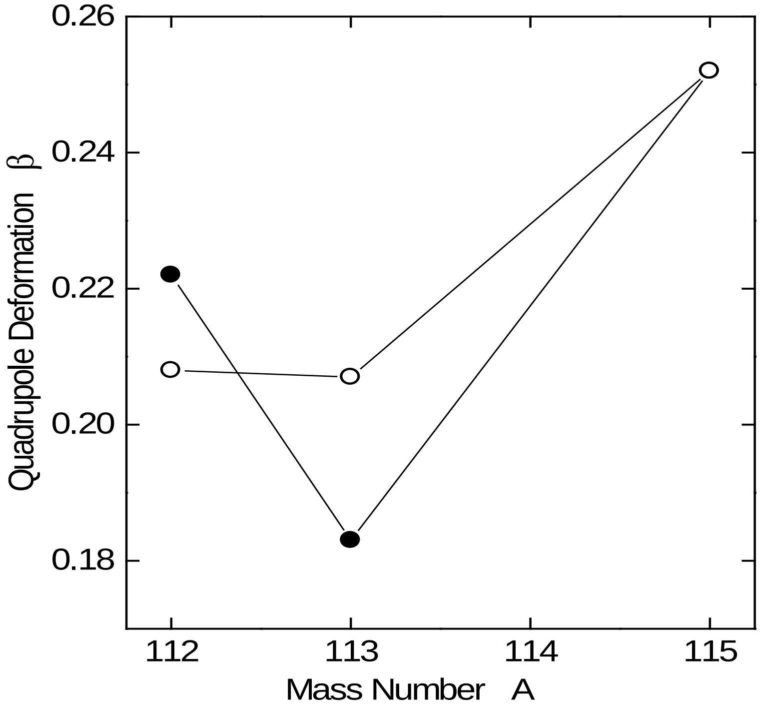

Fig.6. Deformation parameter as a function of Mass Number of Pm isotopes

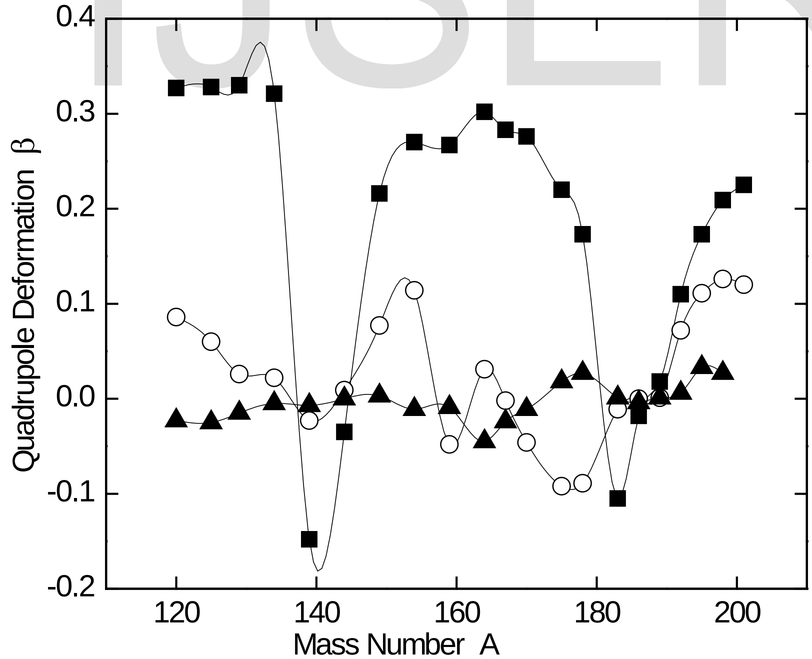

Fig. 7. Quadrupole Deformation values calculated (dark circle) for proton emitters of Cs isotopes in comparision with Moller et al values(open circle)

Fig. 7 shows the calculated deformation values (dark circles) as a function of mass number for Cs isotopes in comparison with Moller values (open circles). The isotope is found to have a shape change from prolate to triaxial for Cs^{112} nuclide and then to have a prolate shape for $Cs^{113,115}$ nuclides. The structural information is obtained using potential energy surface calculation. The potential energy surface plot for Cs nucleus is shown in Fig. 8. The minimum at ground state spin reveals that Cs has got a triaxial shape with normal deformation.

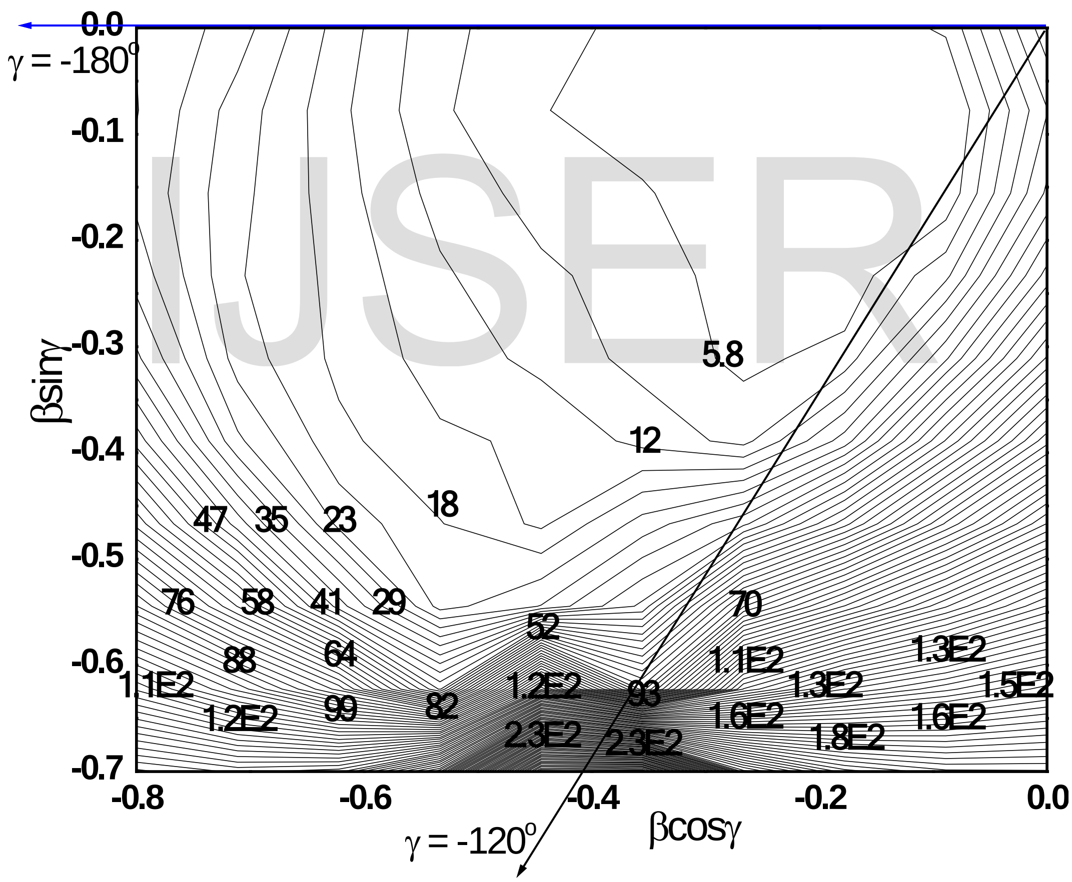

Fig. 8. Potential Energy Surface plot for Cs^{112}

4.1 Summary and Conclusion

The most important observations made in this paper can be summarized as follows: For nuclei, far from stability significant shape deformation in their ground state is observed. This is reflected in the calculations of quadrupole deformation parameter. Nuclei with a few extra nucleons outside a double closed core have positive charge distribution preferring prolate shape. It is obvious from the above discussion, that the meaning of deformation in the proton radioactivity is rather complex. At the same time nuclei which are in short of few nucleons to complete double closed orbits have negative charge distribution preferring oblate shape. It is conceivable those chosen nuclei live beyond the proton drip line but can be observed experimentally. The discovery of proton emitters in the elements of La, Pr, Pm, Tb with deformation effects would provide a wealth of nuclear structure information on nuclei with wide range of deformation. The theoretical models used to calculate the properties of proton emitters should consider a deformed shape for the parent nucleus.

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